Evolutionary game theory for biologists

2013 August 12
Baltimore, MD

We want to demonstrate that a model can be used to analyze data without direct use of a personal computer.

Sample problem

1. This problem refers to the following situation and the plots/blank graph paper enclosed: The number, *C*, of copper cells and the number, *D*, of denim cells in a particular container are hypothesized to vary in time in a way consistent with a basic evolutionary game theoretic model with pairwise interactions. The absolute populations are thought to be described by the differential equations

$$\frac{dC}{dt} = (Rp_C + Sp_D)C \qquad \frac{dD}{dt} = (Tp_C + Pp_D)D$$

with $p_C := C/(C + D)$ and $p_D := D/(C + D)$ denoting the copper and denim population fractions, respectively.

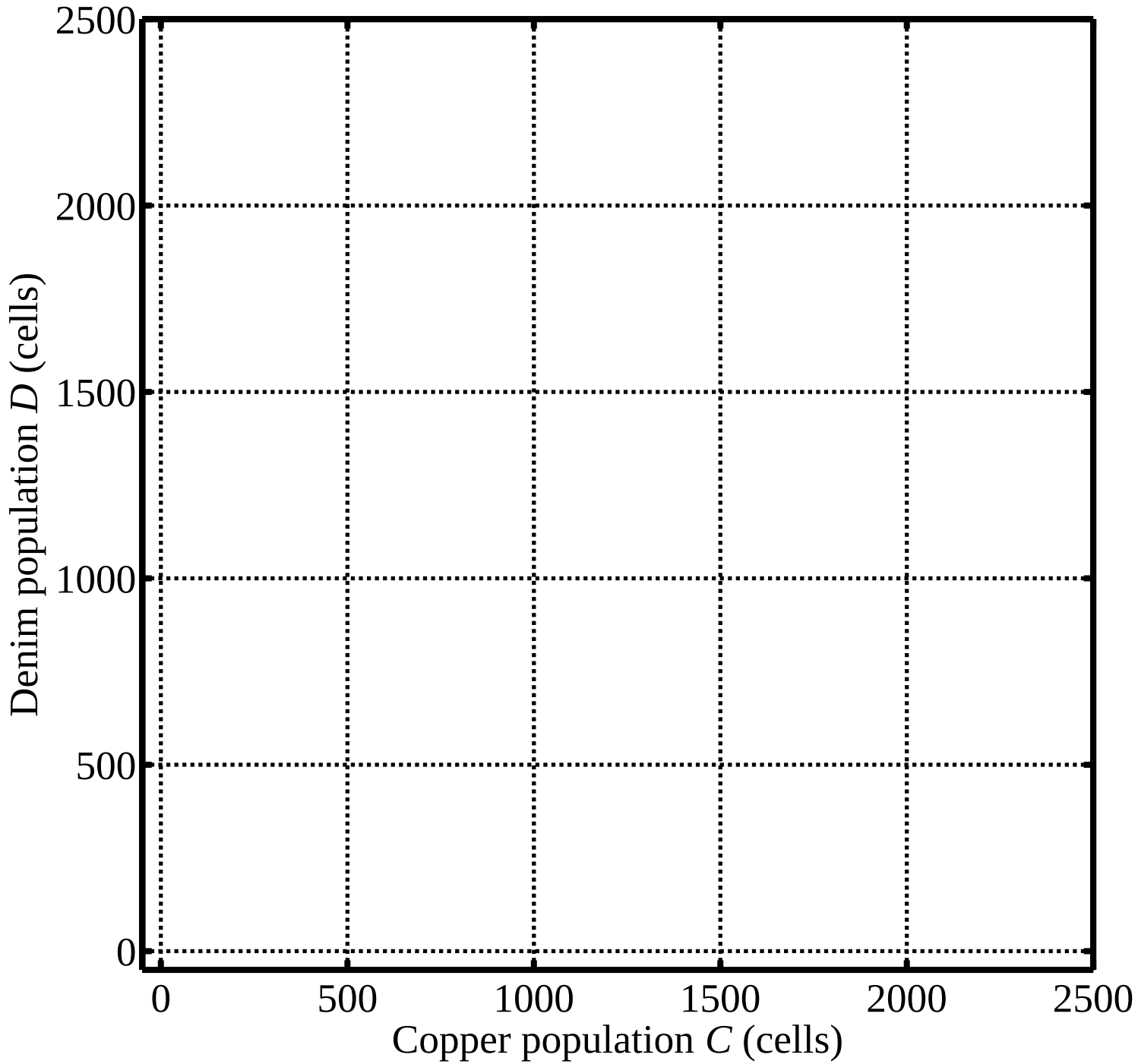
(a) Estimate the parameters *T*, *R*, *P*, and *S*. Express your answers in units of day^{-1}.

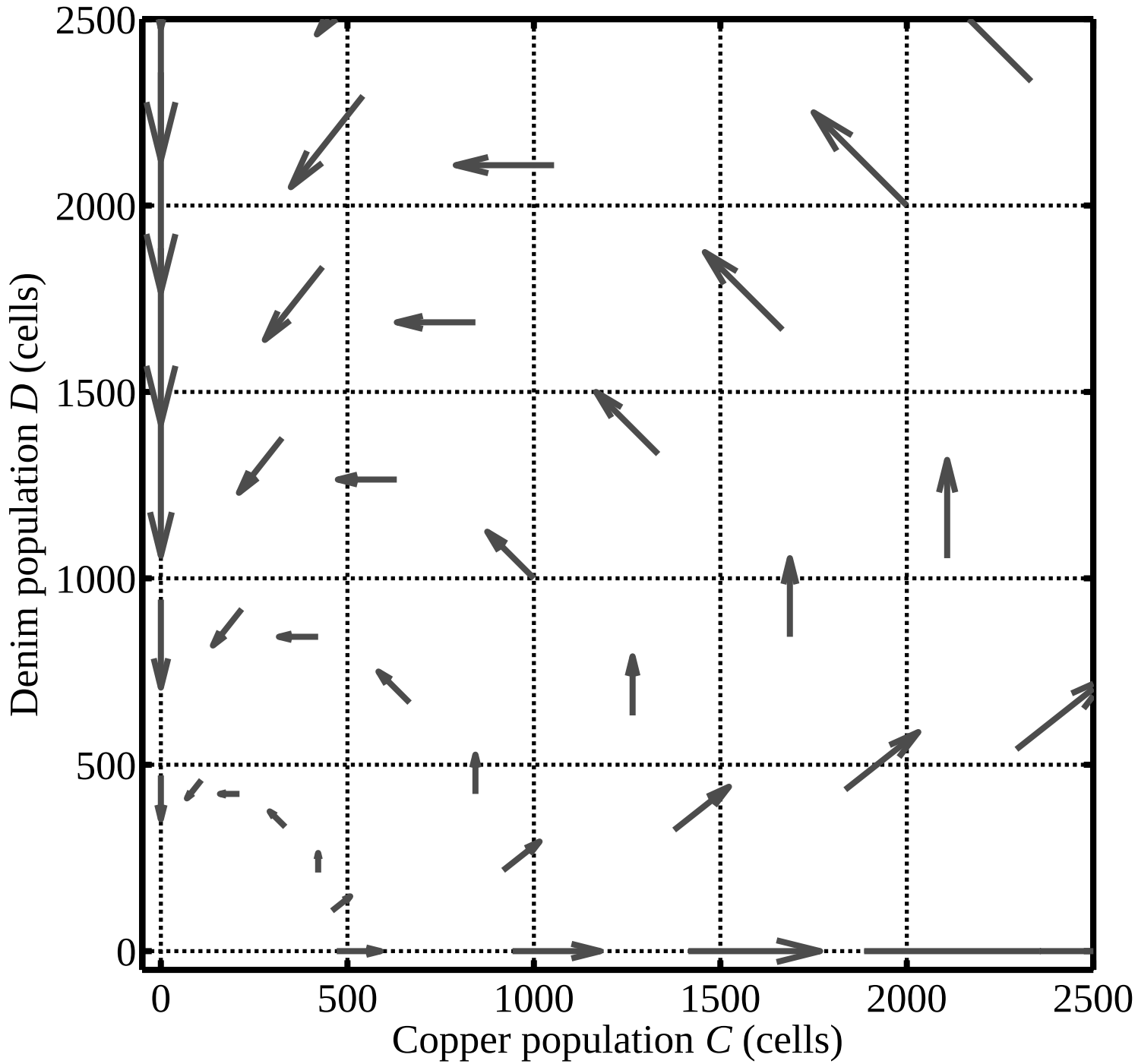
T = _____ *R* = _____ *P* = _____ *S* = _____

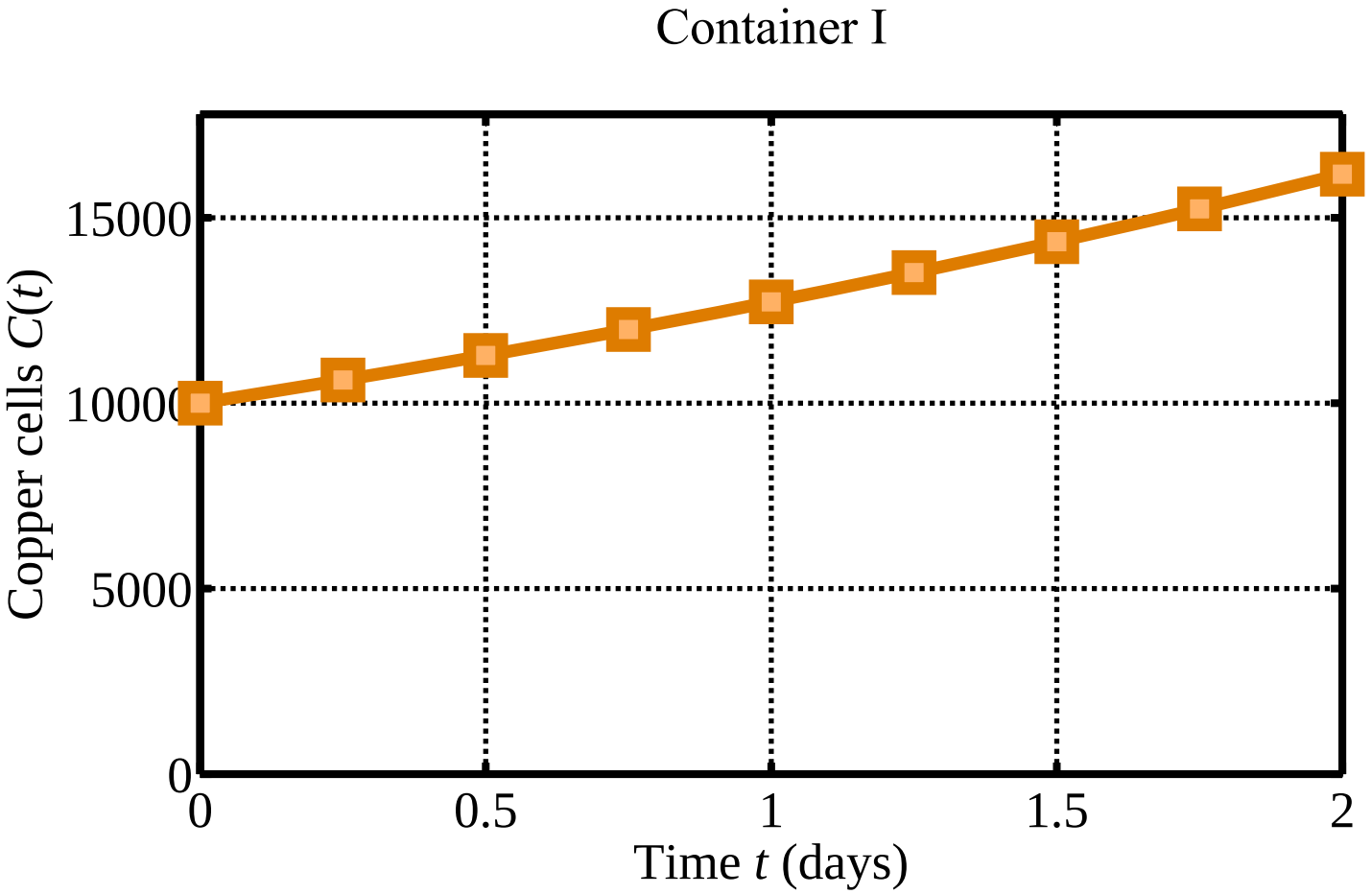
(b) Draw quivers on the provided sheet of graph paper to approximate how much the copper and denim subpopulations would change over the course of a day, starting from various initial subpopulation sizes.

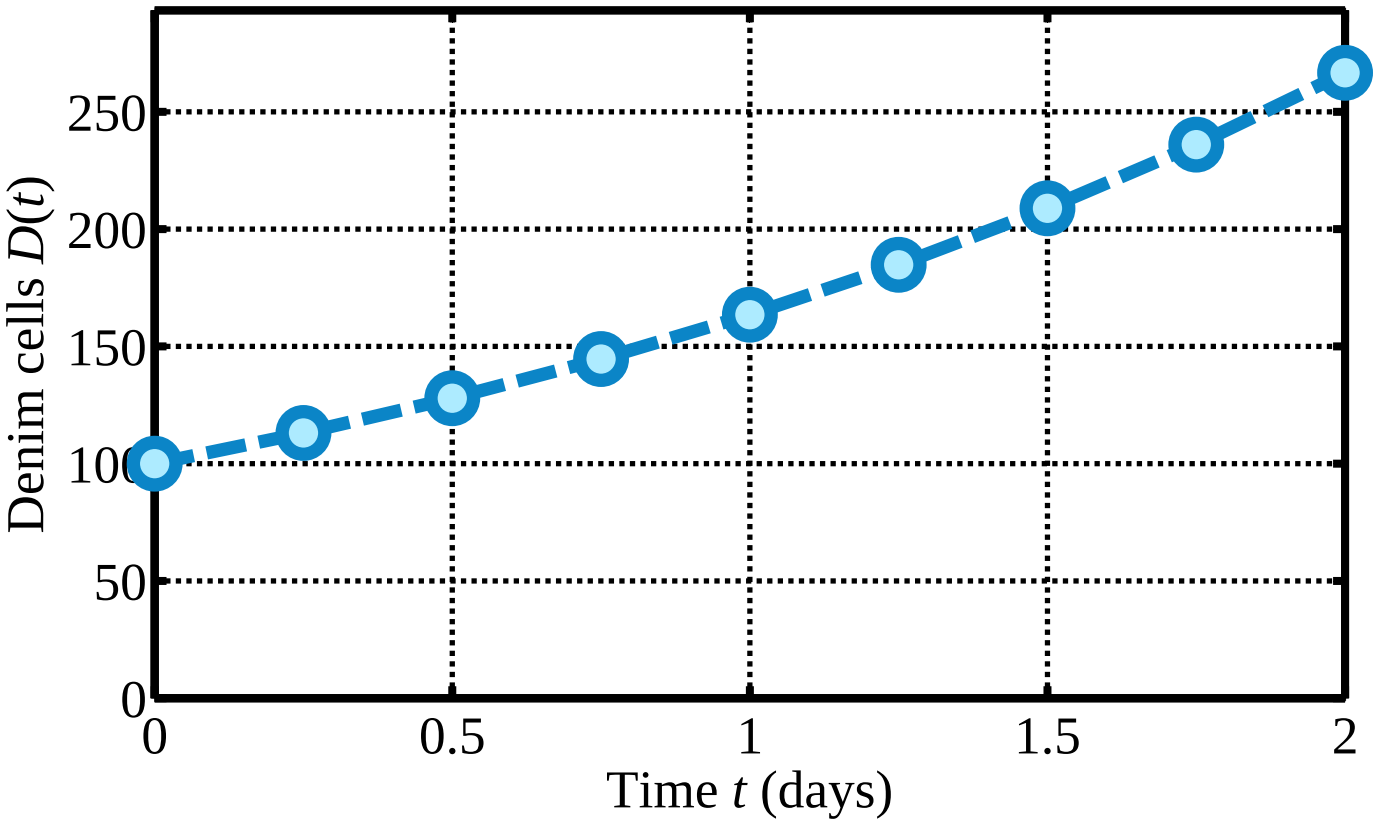
(c) Represent the data from container III as a phase path in the phase plane you have just sketched. Is the trajectory consistent with the quiver field in direction and magnitude?

(d) Explain how you have trained and validated the model in this problem.

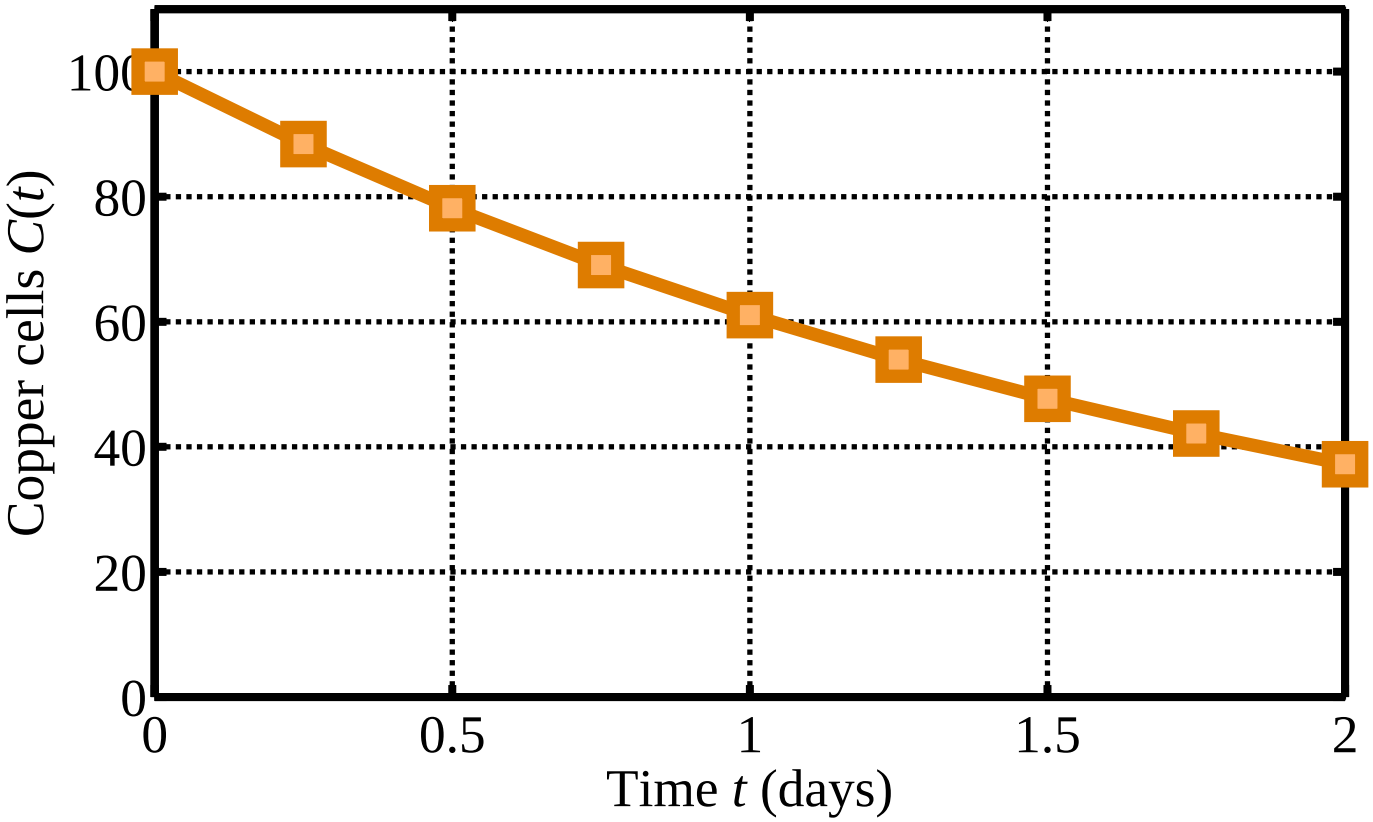

2500
2000
1500
1000
500
0
Denim population D (cells)
0
500
1000
1500
2000
2500
Copper population C (cells)

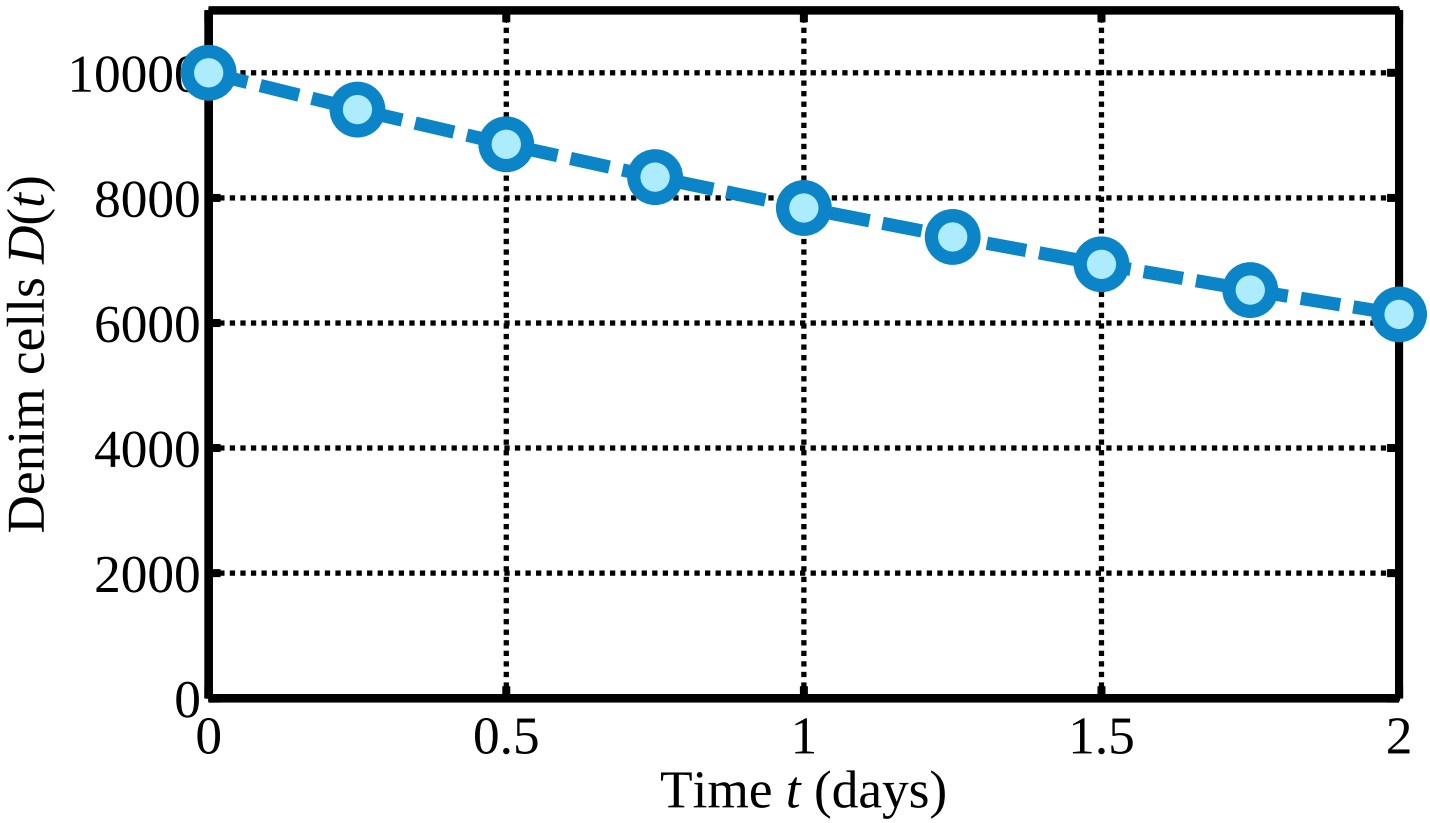

2500
2000
1500
1000
500
0
Denim population D (cells)
0
500
1000
1500
2000
2500
Copper population C (cells)

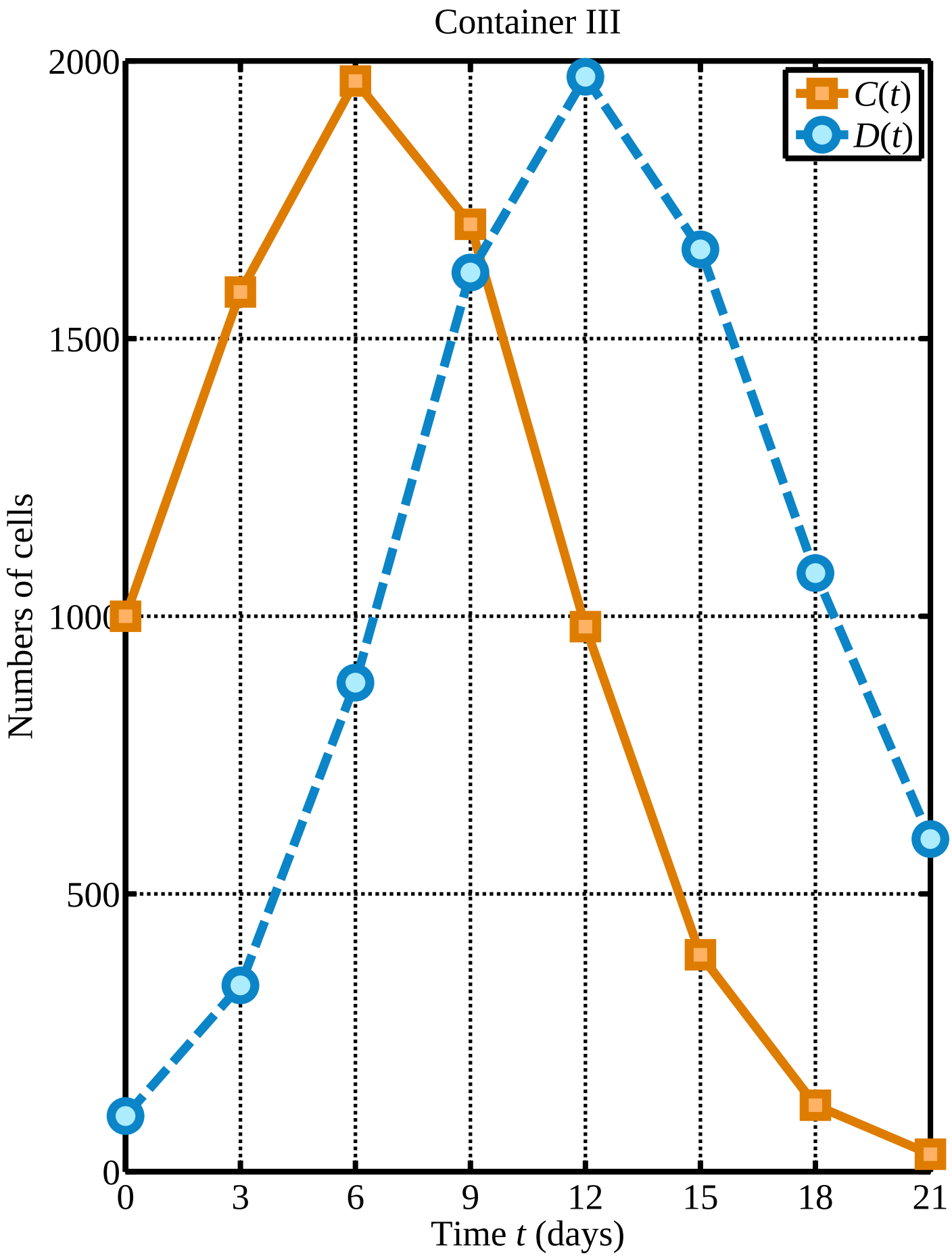

Container I
Copper cells $C(t)$
15000
10000
5000
0
0
0.5
1
1.5
2
Time t (days)


Denim cells $D(t)$
250
200
150
100
50
0
0
0.5
1
1.5
2
Time t (days)

Container II






Container III
2000
1500
1000
500
0
Numbers of cells
0
3
6
9
12
15
18
21
Time t (days)
C(t)
D(t)